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REVIEWS

Hydrodynamic Instabilities and the Transition to Turbulence. Topics in Applied Physics, vol. 45. Edited by H. L. SWINNEY and J. P. GOLLUB. Springer, 1981. 292 pp. $50.40.

Transition and Turbulence. Edited by R. E. MEYER. Academic, 1981. 245 pp. $15.50.

Chaos and Order in Nature. Edited by H. HAKEN. Springer, 1981. 275 pp. $28.00.

The appearance of three books in one year on the topic of transition to turbulence (and closely related phenomena) is a remarkable occurrence which reflects the recent explosion of interest in bifurcation theory applied to dynamical systems having a finite number of degrees of freedom, and the possible relevance of this theory to the behaviour of fluid systems near to critical values of the governing stability parameters. The bridge between fluid systems (with infinite freedom) and dynamical systems (with finite freedom) is provided by 'centre-manifold theory' which provides some justification for focusing attention on the behaviour of a small number of interacting modes of excitation when the fluid system is in the transcritical stability regime; other modes are present, but they would be damped in isolation, and they play a subsidiary role – the so-called 'slave variables'.

Fluid-dynamicists have of course been feeling their way in a heuristic manner towards such descriptions for some time, the most celebrated and widely studied 'centre manifold' being that considered by E. Lorenz in 1963 ('Deterministic non-periodic flow' *J. Atmos. Sci.* vol. 20, p. 130). The Lorenz system falls within the class

$$\dot{x}_i = a_{ij}x_j + b_{ijk}x_j x_k \quad (i = 1, 2, 3), \tag{1}$$

exhibiting quadratic nonlinearity; for certain ranges of parameters defining the coefficients a_{ij}, b_{ijk}, the solutions have a random character and the limit set ($t \to \infty$) of the representive point (x_1, x_2, x_3) in phase space is a 'strange attractor' – i.e. (roughly speaking!) a surface folded on itself infinitely often in such a way that a line normal to it intersects it in a Cantor set. The solution is then sensitive to initial conditions in the sense that trajectories that start close to each other in the phase space do not remain close as time progresses. This type of behaviour has for long been recognized as one of the hallmarks of turbulence; and it is this property that led Ruelle & Takens in 1971 ('On the nature of turbulence' *Commun. Math. Phys.* vol. 20, p. 167) to conjecture that the random behaviour familiar in turbulent fluid systems is intimately related to the strange-attractor behaviour that appears in the superficially much simpler nonlinear dynamical systems.

An alternative view of the transition to turbulence had been proposed in 1944 by Landau ('On the problem of turbulence' *C.R. Acad. Sci. U.S.S.R.* vol. 44, p. 311); this was that, as the Reynolds number increases for a given flow, new modes of instability appear (through Hopf bifurcations) at a succession of critical values $R_{c1}, R_{c2}, R_{c3}, \ldots$, the resulting flow being quasi-periodic as a function of t, and therefore relatively *insensitive* to initial conditions. Although given great publicity through the celebrated text *Fluid Mechanics* (Landau & Lifshitz, Pergamon, 1959), it seems fair to say that this picture of transition was never really widely accepted among turbulence researchers, and this essentially for the reason put forward by Monin & Yaglom (*Statistical Hydrodynamics*, vol. 1, Nauka, Moscow, 1965): 'the most important defect of Landau's theory is that, so far, it has not been verified by direct calculations...'.

The merit of strange-attractor theory is not so much that it tells us that random behaviour can arise out of deterministic systems, for in the context of the Navier–Stokes equations that is a fact of life that has been accepted for decades, but rather that this behaviour can occur in systems with as few as 3 degrees of freedom, and that we therefore have a new handle, as it were, on the problem of *incipience* of random behaviour, which is what *transition* to turbulence is all about. The fact that, in low-order systems such as (1), bifurcation to a strange attractor can occur after two or three bifurcations of classical type is moreover strongly suggestive in the context of (*a*) Couette flow between concentric cylinders, with the inner one rotating, and (*b*) Bénard convection in a horizontal layer heated from below, which both exhibit a short sequence of deterministic instability modes as the relevant stability parameter increases before a transition to turbulence occurs.

The first of the three books listed above (*Hydrodynamic Instabilities and the Transition to Turbulence*) contains a collection of nine specially commissioned articles:

1. Introduction, by H. L. Swinney & J. P. Gollub;
2. Strange Attractors and Turbulence, by O. E. Lanford;
3. Hydrodynamic Stability and Bifurcation, by D. D. Joseph;
4. Chaotic Behaviour and Fluid Dynamics, by J. A. Yorke & E. D. Yorke;
5. Transition to Turbulence in Rayleigh–Bénard Convection, by F. H. Busse;
6. Instabilities and Transition in flow between Concentric Rotating Cylinders, by R. C. Di Prima & H. L. Swinney;
7. Shear Flow Instabilities and Transition, by S. A. Maslowe;
8. Instabilities in Geophysical Fluid Dynamic Systems, by D. J. Tritton & P. A. Davies;
9. Instabilities and Chaos in Nonhydrodynamic Systems, by J. M. Guchenheimer.

The range and quality of these articles are outstanding, and the collection provides a fascinating introduction both to the fluid mechanics of transition and (through chapters 2, 3, 4 and 9) to the behaviour of low-order dynamical systems that are related either directly, or by analogy, with the transition problem. The articles have been carefully organized and edited and the general standard of production is excellent. My sole criticism relates to the references at the end of each chapter, which are arranged not alphabetically, but in the order of first appearance – a mildly irritating system when, as so often, one is looking for a particular reference from memory of an author's name; and some of these lists are long – there are 119 references at the end of chapter 5 alone!

The second book on the list (*Transition and Turbulence*) contains the Proceedings of a symposium conducted by the Mathematics Research Center of the University of Wisconsin in October 1980. This contains twelve papers by authoritative contributors, reproduced from camera-ready copy. Transition in rotating Couette flow, Bénard layer, jets, pipes and channels and boundary layers are all represented, plus three papers on coherent structures. The meeting was evidently well-balanced and stimulating and these Proceedings make a useful and welcome addition to the literature.

The third volume (*Chaos and Order in Nature*) contains the invited papers of an international symposium on Synergetics (which 'deals with systems composed of many subsystems') held at Schloss Elmau, Bavaria, in April 1981. Only 5 of the 26 papers deal with fluid mechanics (yet again mainly the rotating Couette problem and the Bénard problem !), but many of the remainder investigate mathematical properties of dynamical systems like (1) which may have a bearing on fluid-mechanical problems. For example, G. R. Sell argues that the Landau conjecture and the Ruelle–Takens conjecture referred to previously are not necessarily incompatible, and that a Hopf–Landau bifurcation can occur in the vicinity of a strange attractor! Again the production is photographic, from camera-ready copy, with associated merits of speed of production and economy (compare the prices of the three volumes under review !). The papers are well grouped, but many lack abstracts, which makes digestion that much more difficult. It is nevertheless a fascinating collection which overlaps a little with the other two volumes, but is on the whole complementary to them.

It seems fair to predict that the current spate of activity in this area will continue for some years, and progress in the understanding of dynamical systems is likely to be rapid. An area that is of equal interest to pure and applied mathematicians is evidently ripe for development, and the scope for fruitful interaction should not be underestimated. These three volumes will provide an admirable means of entry to the field.

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